CRUDE REGISTRATION OF RANGE IMAGES THROUGH MATCHING OF THEIR SIMPLIFIED MESHES

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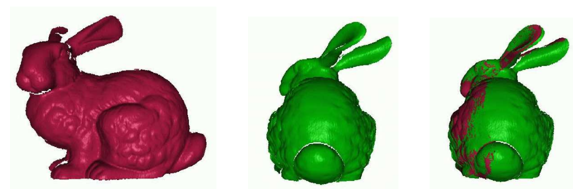
ABSTRACT
For constructing a tri-dimensional model from range images, they must be registered, i.e., it must be put together into the same coordinate system. There are good algorithms available to perform this, but only after the range images have been brought into positions that are close to the true ones with respect to an arbitrarily chosen reference system. These initial estimates are often provided manually. This paper deals with a technique to compute them automatically. We present a way to determine parameter values that are necessary to control the control flow of a simplified mesh based crude registration, which is characterized by two novel features: drastic data reduction and an expressive shape descriptor.

KEY WORDS
registration, correspondence, geometric descriptor.

1 Introduction

Three dimensional modeling has many applications in computer graphics, virtual reality, medical science, reverse engineering and robotics. Various techniques including structured light and laser range finders are used for acquiring range images of an object. A range image (or a view) however is not sufficient to completely model a free-form object [1] due to self-occlusions. Multiple views of the object must be acquired in order to cover the entire surface of the object. These views must then be registered in a common coordinate basis. Figure 1 illustrates the registration of two range images, the target $S-$image and the reference $D-$image, acquired from two different points-of-view defined in proper coordinate references.

Iterative algorithms, such as iterative closest point (ICP) [1, 14, 9], can determine the relative poses accurately, provided an initial, approximate pose is known. These methods perform an optimization and as such are sensitive to local extrema. When the initial poses are very different they might not converge to the correct solution. We will refer to the determination of this approximate pose as crude registration. The refinement of this pose by an algorithm such as ICP will be referred to as fine registration. Whereas fine registration can be automated, often the crude registration is done manually or through knowledge of the relative viewpoints from which the surfaces were recorded. Manual crude registration can for example be done by letting a user position the range images interactively or by having the user mark some corresponding points on them. This can become a tedious job when many range images have to be positioned or when they have no clear features that can be matched easily by a human. The primary contribution of our work is the automation of the crude registration.



(a) (b) (c)

Figure 1. Range images of a bunny: (a) S-image; (b) D-image; (c) integrated 3D- model.

The registration task could be resolved (1) by detecting corresponding points which are points on two different views that correspond to the same point on the object, and (2) by using these correspondences for deriving the rigid transformation (rotation and translation) that aligns the views. The obvious challenge here is automatically resolving the correspondence problem

The traditional way to solve the correspondence problem is by matching descriptors that two differing images have in common. There are three steps to the descriptor matching process: detection, description and matching. Detection refers to capability of extract a reasonable number of samples from which we can construct matchable descriptors. Because that a range image usually has hundreds of thousands samples, there are two approaches for detection: random selection and special points extraction. Both are tentatives to avoid the use of complete sample set which results in excessive number of descriptors, making the algorithm inefficient. On other hand, the selection of few or very restricted sample subset can result in loss of information which can be relevant for success in matching. This means that there is a trade-off between preservation of relevant information of surface and efficiency of the algorithm. In [7], we proposed a method that leads to a drastic reduction of the input data, without missing the relevant features of the original dense data.

The descriptions of the selected samples must be distinctive enough to filter out wrong matches while remaining invariant to rigid transformations as rotation and translation. A descriptor with low discriminative capability will result in multiple ambiguous matches. In [7], we presented, associated with each selected sample, a descriptive spatial trihedron that encodes spatial angular and metric information of the surface. It hugely reduces the volume of potential matchable candidates.

Finally, the matching step consists in obtaining the transformation $\mathcal{T}$ from the remaining matches. This is done by two steps: generation of candidate transformations through matching of pairs of descriptors and by filtering out the false positive transformations. The classic procedure for candidate evaluating is global evaluation of errors, i.e., the transformation must be applied and the distance between points of both images calculated. The efficiency of this procedure depends on the data volume size (first step) and the discriminating capability of the descriptors (second step). On the other hand, the correctness of algorithm is ensured by verification of all the candidate transformation for avoiding that the correct transformation is excluded. In [7], we showed that after carefully reducing the search space, it is feasible to consider all the candidate transformations.

The main drawback of the procedure presented in [7] is, however, that it needs several control parameters that depend on the expert knowledge, such as threshold for simplification error and the appropriate size for shape descriptor. In this paper, we give some working heuristics for determining such parameter values. Moreover, we propose to encode two more geometrical properties, namely the minimal and maximal curvatures, in the shape descriptor.

2 Correlated Works

Next, we present three of the most widely used method for automatic registration of two partially range images.

Matching oriented points uses the spin image which relates the neighboring points of a sample to its normal vector by generating a 2D histogram whose coordinates are distance along the sample's normal vector and radial distance from the normal vector, with the sample location as the origin [5]. The spin images of randomly selected samples from one view are matched with the spin images of all the samples of the second view using correlation coefficients. One of the problems with spin images is that information is lost in the projection to two dimensions which leads to an inability to discriminate between samples that otherwise should be considered different. An example would be two features that are mirror images of each other. Other problem is that spin images of close points on the same view are very similar. Both situations make that spin image matching results in many ambiguous correspondences which must be processed through a number of filtration stages to prune out incorrect ones. Even after these filtration stages the algorithm is left with a large number of geometrically false positive matches which must be verified individually, making the algorithm inefficient even for range images of a reasonable size. When only some candidate transformations are verified, the algorithm can generate a false positive transformation as result, in according to Planitz et al. [2]. The method proposed in [7] not only has less data volume but is suitable for matching features that are mirror images, as well. The global evaluation of all the reduced number of remaining transformations allows the correct filtration of the false positive transformations.

The RANSAC-based DARCES (Random Sample Consensus-based Data Aligned Rigidity Constrained Exhaustive Search) is a technique that selects three non-collinear points, a primary, a secondary and an auxiliary point, from the $S-$image at random [3]. Then it hypothesizes a point in $D-$image to be the corresponding point of the primary point and searches for the corresponding points of the secondary and auxiliary points while observing the rigidity constraint. In Figure 2(a) a triangle formed from the three control points (S_p, S_s, S_a). S_q is the orthogonal projection of S_a to line S_pS_s and d_{qa} is the orthogonal distance from it. The search region for the corresponding point to S_s is a sphere, as illustrates Figure 2(b). Figure 2(c) shows the corresponding point M_q to S_q in the model. The search for the corresponding point to S_a is restricted to the circle depicted in Figure 2(c).

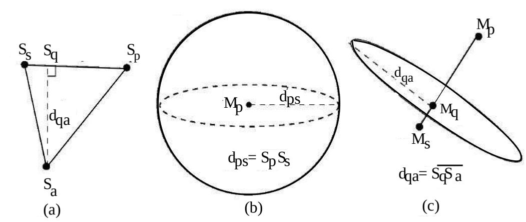


Figure 2. Search for correspondences.

For every possible set of three correspondences a candidate transformation is calculated, applied and verified. This process continues until the overlapping level, after of application of a candidate transformation, is greater that a predefined threshold. Applying the rigidity constraint considerably reduces the search space. However, the number of possible combinations is still excessive to make the algorithm feasible. Instead of a planar structure (triangle), a spatial structure is used in [7]. It is more distinguishable making fewer matchable descriptors to be selected. Additionally, the reduced data volume makes feasible to scan all of remaining matches for getting the best one.

Bitangent curve matching is a technique based in feature matching. Bitangent curves, which are curves that share the same tangent plane, are chosen as an invariant feature [15]. The end points of these curves give four points on one surface which are used as descriptors. The distance function of the bitangent point pairs in terms of the bitangent curve arclength is used for matching. The candidate transformations obtained from matching descriptors are applied and verified by checking the adjustability of bitangent curves. Some inconveniences are detected in this technique. Bitangent curves are global features of the surface and may not be fully contained inside the region of overlap. Hence, one of the end points of the bitangent curves may lie outside the region of overlap causing an incorrect correspondence. Moreover, bitangent curves are very special features and it is not ensured that they are present in every image. The reduction procedure presented in [7] ensures that enough quantity of samples be preserved; hence, a matching descriptor can always be built from each sample without neither consideration of geometric form nor existence of relevant features. Only a minimal and maximal estimates of percent of $S-$image that is in overlapping region are required.

Because that the objective of a crude registration is to approximately align two images on the basis of the overlapped region, we only need the overall shape of image. In fact, it will be verified in Section 7, excessively detailed models do not necessarily deliver more accurate result. Following this reasoning and for sake of efficiency, we come to the idea to use the samples of decimated meshes rather than dense meshes to find the rigid transformation $\mathcal{T}$ that overlays the S-image over the D-image. Based on the hypothesis that simplified meshes encapsulate enough of the geometrical characteristics of the dense ones, we devise a robust crude registration procedure [7]. Nevertheless, it requires several control parameters that are still set on the time-consuming trial-and-error basis.

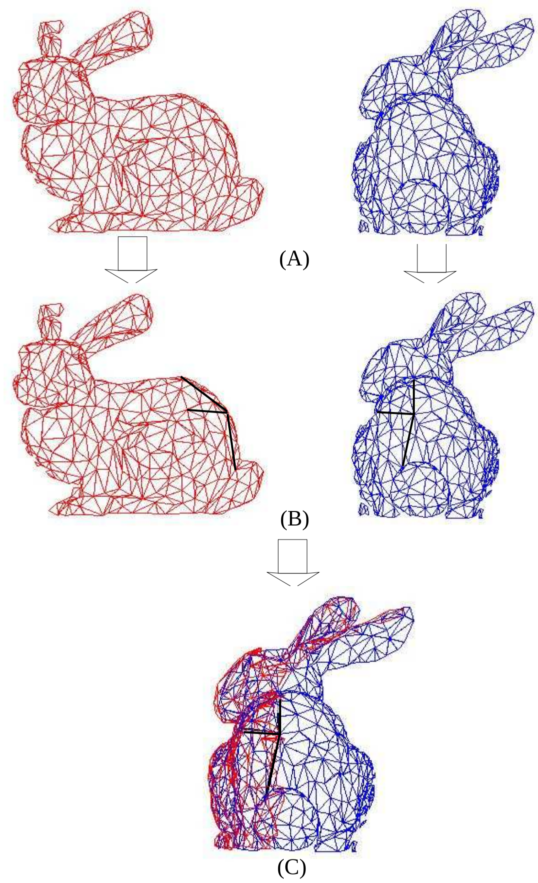


Figure 3. (a)Simplified meshes (b) trihedrons and (c) matching result.

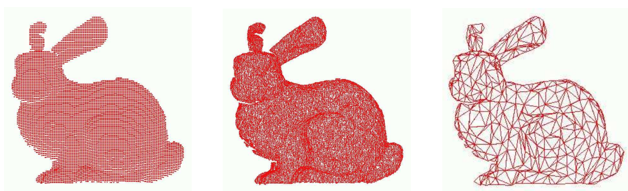
3 Overview of the Approach

In 3-D computer vision, the amount of computation is often proportional to the number of data points [5]. In the automatic registration task, the computational effort also depends on the expressive power of the matching descriptor, once a such descriptor will avoid to generate multiple ambiguous matches. So, to minimize computation, our algorithm should minimize the number of sample in the range image without loss the relevant features and to propose a distinctive descriptor. Our proposal is characterized by two novel features related to these aspects: drastic data reduction and expressive shape descriptor. It basically consists in three steps: (a) reduction, (b) descriptor construction, and (c) matching.

First, in the reduction step, we create two simplified triangular meshes $S-$mesh and $D-$mesh for $S-$image and $D-$image respectively (Figures 3(a)). Each simplified triangular mesh is obtained from a dense triangular mesh that interpolates all samples of range image. As the goal is matching, the simplification does not consider only reduction with shape preservation, but also a distribution of the vertices over the mesh in an adequate density. In second step, descriptors called spatial trihedrons are then constructed from vertices of simplified meshes, as illustrated in Figures 3(b). A trihedron is determinated from four samples: an origin and three more vertices far enough apart, so that it is able of provide geometrical information of a large spatial neighborhood. The local minimal e maximal curvatures are estimated and also encoded in the trihedron. These several geometrical informations makes trihedron a expressive descriptor. Once this descriptor construction process has been completed for each vertice of $S-$mesh, we can find descriptors on $D-$mesh. The correspondence vertices are search for observing the rigidity constraint. Once we have a set of correspondences, represented by our descriptors pairs, we can compute a set of candidate transformations and chosen from them the transformation $\mathcal{T}$ that best aligns the vertices of the two meshes. Figure 3(c) illustrates the matching result of two simplified meshes.

4 Control on Mesh Simplification

The range images, in the form of point clouds (Figure 4(a)), are converted into triangular meshes (Figure 4(b)). To build a mesh, we create triangles from four samples of a range image that are in adjacent rows and columns as described in [14]. The shortest of the two diagonals between the samples is used to create two triangles. A distance threshold is set in order to prevent joining some range points that should not in fact be connected. Let s be the maximum distance between adjacent range points when we flatten the range image, we take the distance threshold be a small multiple of this sampling distance, i.e., $5s$. To reduce the mesh, a mesh simplification algorithm must be applied resulting in a simplified mesh (Figure 4(c)).



(a) (b) (c)

Figure 4. (a) Range Image Bunny; (b) Dense Mesh; (c) Simplified Mesh.

Traditionally, the simplification algorithms were developed inside of computer graphics purposes. There, the main motivation for mesh simplification is to reduce the number of faces describing an object, while preserving the shape of the object, so that the object can be rendered as fast as possible. However, for registration purpose, the algorithm also must establish a adequate level of detail that guarantees acceptable sample density in order to not miss the minimal number of samples necessary on the overlapping region. So, for our purpose the simplification algorithm must be able of

- preserving shape,
- minimizing number of vertices, and
- the sample density (number of samples per unit area) being adequate for guaranteeing the existence of correspondences in overlapping areas.

We choose the efficient QSLIM mesh simplification algorithm [6], that simplifies the mesh while preserving maximum amount of geometric variation on its surface, and adapted it for evaluating our proposal in practice.

In this section we define an approximation error ϵ which will establish the resolution of the meshes $S-$mesh and $D-$mesh. QSLIM reduces the sample density on smooth regions and increases it on regions with prominent features. For establishing an unique approximation error ϵ for any geometry, we introduce a smooth parameter $\%smooth$ in the calculation of ϵ. The smooth parameter of a range image can be easily defined by estimating the minimal and maximal curvature of the dense mesh vertices. The estimation of these curvatures is done with use of the Rusinkiewicz algorithm [10]. On the basis of the curvatures, we categorize the vertices as planar points (p), parabolic (pa), elliptical (e) and hyperbolic (h), and define the $\%smooth$ parameter as follows

$$\%smooth = \frac{p + pa + e}{p + pa + e + h} * 100.$$

In addition, we perceive that a reduction must be proportional to sampling density of the range images, then we also introduce the sampling frequency parameter $s * A$, where s is the sampling distance and $A = max(Dx, Dy)$, where Dx and Dy correspond to horizontal and vertical length of the bounding box containing the range image, respectively. So, we arrive to following formula for approximating error

$$\epsilon = \frac{0,15 * s * A}{\%smooth}$$

that guarantees acceptable sample density in our exhaustive tests with images available in the repositories [8, 13, 12].

5 Control on Descriptor's Size

Registration of range images is based on partial matching task which consists of matching sub-parts or regions. The parts that are matched can be any sub-shape of a larger shape in any orientation and must satisfy certain similarity measures. The challenge is to choose distinguishable parts to alleviate the task of search of matches, making it more discriminative. We define a descriptor whose descriptive power is sufficiently high and effective due to the following reasons:

- it is a spatial structure rather than a planar one as triangles presented in [3]
- it incorporates a regional context of the shape rather than only a local one, and
- it incorporates local geometric information for guiding the punctual matching.

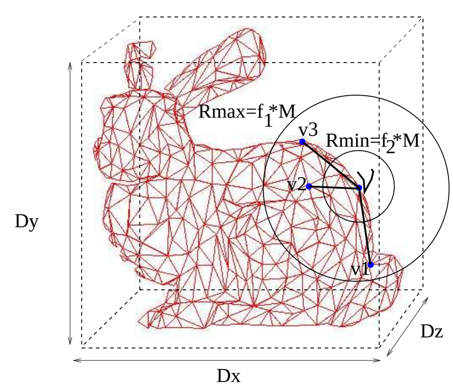


Figure 5. Spatial Trihedron

We propose as descriptor a spatial trihedron which consists of three linear independent vectors preferably orthogonal, whose origin $\mathcal{V}$ and extreme points of its axes are vertices $v_i, i = 1, 2, 3$ of the simplified $S-$mesh as view in Figure 5. To set the distance d_i of vertices to the origin $\mathcal{V}$, we consider the trade-off between maximal coverage and permanence inside the region of overlap. A trihedron that incorporates a large region is likely to be more distinguishable. However, such a trihedron can lie outside the region of overlap. To address these concerns, we develop a conciliatory way of restrict the size of trihedron to a estimative of percentage that the overlapping is of the range image. So, distances d_i will be

$$f_1 * M \leq d_i \leq f_2 * M, \quad f_1 < f_2$$

where $M = (D_x + D_y + D_z)/3$ with Dx,Dy and D_z corresponding to horizontal, vertical and deep length of the bounding box containing the range image (Figure 5). The f_1 and f_2 values should be provided by a user as the minimal and maximal estimative of percentage that region of overlap represents in relation to complete range image. Nevertheless, we believe that this is an easy task. When an appropriate image visualization is provided, we may rely on the user's visual intelligence to do such estimates.

6 More Geometrical Properties in Matching Procedure

The correlated works point to a trade-off between preservation of relevant information of surface and efficiency of the algorithm. On a side, a dense set of descriptors is likely not to miss the best match and, on the other side, an exhaustive approach would be overly costly due to huge combinatorial complexity in the matching. In this section, we briefly describe our exhaustive matching procedure in order to show how the minimal and maximal curvatures are integrated in the procedure.

We define a trihedron in each vertex of $S-$mesh and exhaustively search for corresponding trihedrons in $D-$mesh. Since the number of vertices is significantly smaller than the number of samples of original range image, we will have a small set of descriptors that allows to reduce the combinatorial complexity of the surface representation. In addition, the descriptive power of the descriptor is able of significantly reduce the combinatorial complexity in the matching. Let $\mathcal{S}$ be a descriptor of $S-$mesh, with origin in vertex $\mathcal{V}$ and extreme vertices $v_i, i = 1, 2, 3$. First, we search for corresponding vertex $\mathcal{C}$ on $D-$mesh to the origin $\mathcal{V}$ de $\mathcal{S}$, by comparing local curvatures. The potential correspondences c_1 and c_2 to v_1 and v_2, respectively, are obtained in the same way as done by DARCES (Figure 2), with an additional restriction: to have matchable local curvatures. Finally, the search for candidate c_3 to v_3 is done by the following procedure: we calculate the point-plane distance d from v_3 to the P plane formed by the vectors $\mathcal{V}v_1$ and $\mathcal{V}v_2$, and also calculate the barycenter coordinates of the orthogonal projection x_2 of vertex v_3 on the P plane in relation to triangle $\mathcal{V}v_1v_2$. Then, the candidate point c_3 will be in a small radius around point p, being $p = x_2 + n_2d$, where x_2 is the corresponding position to x_1 on triangle $\mathcal{C}c_1c_2$ and n_2 is the normal vector to plane formed by the vectors $\mathcal{C}c_1$ and $\mathcal{C}c_2$ (Figure 6). From each pair of matches $\mathcal{V}v_1v_2v_3$ and $\mathcal{C}c_1c_2c_3$, we may compute one candidate alignment transformation TS which must be evaluated in terms of their matching power. The best solution $\mathcal{T}$ is the one that best aligns the the $S-$mesh over the $D-$mesh.

Images	Samples	Vertices	%Reduction	f_1	f_2
FROG set	10297	812	92,11	0.3	0.6
DINO set	14286	770	94,61	0.5	0.7
BUNNY set	35647	400	98,88	0.0	0.3

Table 1. Data volume reduction

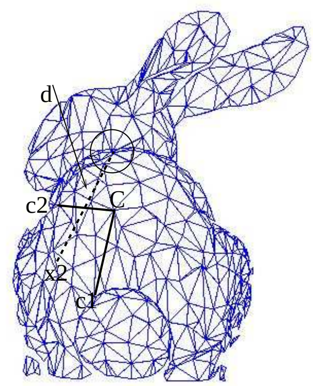


Figure 6. Fourth Vertex Matching

7 Experimental Results

We selected two test sets with 6 range images in each: DINO taken from the repository [13] and FROG from [8]; and one test set with 4 range images, BUNNY from [12]. One set of each repository was chosen for validating our algorithm from several situations (Figures 7 and 8).

After applying the volume data reduction scheme of the Section 4, we observe from Table 1 that, in average, there was a reduction factor of 95% with respect to the original data. Despite this drastic reduction, our algorithm will be still able to deliver a coarse alignment transformation that makes the ICP algorithm converges in few iterations as commented latter.

Figures 9,10,11 present the visual results of our proposed procedure applied on all the possible pairs of range images of each set presented in Figures 7 and 8. The results show that satisfactory crude registration between all the surface pairs is achieved. Each pair of aligned meshes show the aligned trihedrons that cover a large region of mesh. Observe that, according to Section 5, the extension

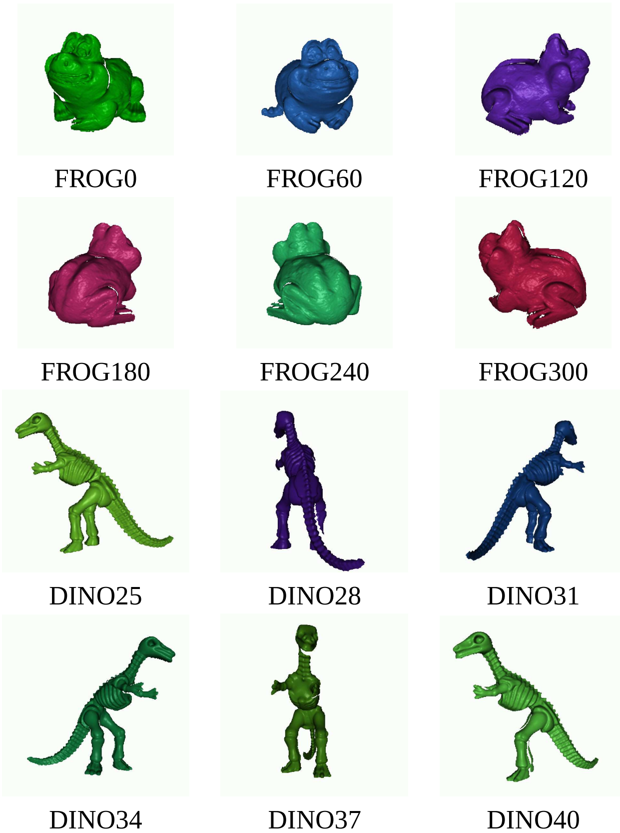


Figure 7. Test Images: FROG [8], DINO [13]

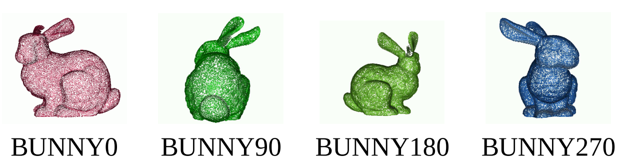


Figure 8. Test Images: BUNNY [12]

of trihedron is limited by estimatives f_1 and f_2 related to size of overlapping region. In Table 1 we present the estimate provide for each set. Note that the overlapping region of BUNNY is considerably small, thus, the automatically generated trihedron is also of smaller size.

According to Planitz et al., we may evaluate quantitatively the performance of our proposal by verifying whether its outcomes lead to correct convergence of the ICP procedure. Using all the transformation matrices as an initial guess, the iterative closest point algorithm (ICP) [1, 4], implemented in Scanalyze [11], iteratively refines them until the mean squared errors between the presumed correspondences are minimized. We used Scanalyze system for evaluating our results. Visually, the outcomes from the Scanalyze is almost indistinguishable from the images shown in Figures 9,10,11. Below each registered pair, we provide the number of iterations that the ICP algorithm needed to converge to the optimal solution. The low number of iterations attests the matching power of coarse meshes.

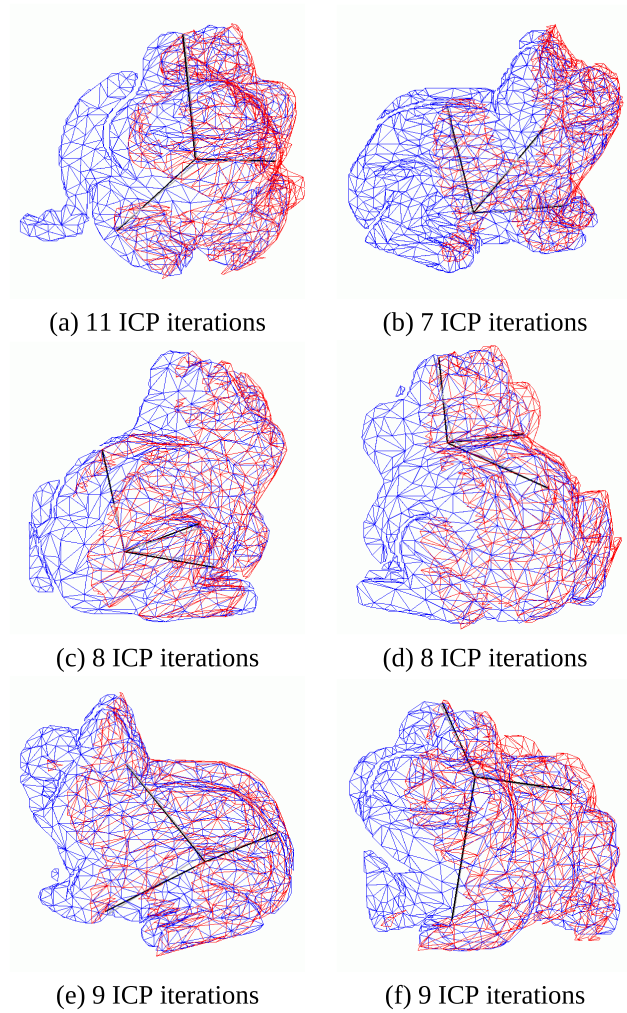


Figure 9. FROG pairs

Other way of evaluating of quality (correctness) of our results is by calculating the difference between transformation real values and ours values. This comparison is possible when the real values are known as is the case of BUNNY [12]. Table 2 presents the pairs of range images Images, the parameter of the real rotation specified in columns $\mathcal{R}_y$ (R) (Rotation about axes y), and the values of the real translation vector given in columns (x (R),y (R),z (R)). In analogous form, parameters obtained by our algorithm are presented in columns $\mathcal{R}_y$ (M) and (x (M),y (M),z (M)) and in columns $\mathcal{R}_y$ (I) and (x (I),y (I),z (I)) we show the parameters after the refinement of our results with use of Scanalyze (fine registration). The quality of our crude alignment can be verified by the small difference between values R and M, and values R and I. In first case, we obtain a angular difference of, in average, 1.4^o and a difference between the values of the translation vector of, in average, 0.002. In second case, the angular difference was of 0.11^o and in values of translation vector we did not have

Images	$\mathcal{R}_y$ (R)	x (R)	y (R)	z (R)	$\mathcal{R}_y$ (M)	x (M)	y (M)	z (M)	$\mathcal{R}_y$ (I)	x (I)	y (I)	z (I)
0-90	90	-0.0001	0.0002	-0.0001	89.9975	-0.0001	0.0004	-0.0007	90.2239	-0.0001	0.0002	-0.0001
90-180	90	-0.0001	0.0001	-0.0001	91.3913	-0.0007	0.0008	-0.0022	90.1095	-0.0001	0.0001	-0.0001
180-270	90	-0.0000	0.0003	0.0000	90.1022	-0.0004	0.0001	0.0009	90.0138	-0.0000	0.0003	0.0000
270-0	90	-0.0001	0.0000	0.0004	93.696	0.0009	0.0035	0.0001	90.0944	-0.0001	0.0000	0.0004

Table 2. Transformation Parameters of BUNNY

difference.

To verify that the proposed reduction does not compromise the precision of results while the gain in time becomes significant, we measure the execution time of our procedure for different reduction level. We use the pair BUNNY0-90 and show the results in Table 3. The time (in seconds) is presented in column Time. The graph of Figure 12 shows the drastic reduction of processing time while the simplification level augments. Observe that values ΔR and ΔD varies inside of a interval of about one degree, this is, parameter values are similar despite of reduction sampling. These values attest that our proposal of control parameter estimation is adequate for data reduction without compromising precision. Note that the reduction of time between the first and last level is of more than 99%.

8 Conclusion

In this paper, we have described a method for estimating the control parameters that are necessary for an automatic crude registration of range images. The crude registration method is based on the use of triangular meshes that captures the essence of the geometry of the original ones, rather than complete image data. Such a mesh allows easiness in constructing discriminative structures. We propose, in addition to our previous work [7], to include the curvature measures in order to make the shape descriptor more discriminant. Experimental results show that our algorithm is accurate and efficient. Numerical data attest that our proposal can reduce, in average, the time more than 99% in relation to spent time when we use a complete sampling.

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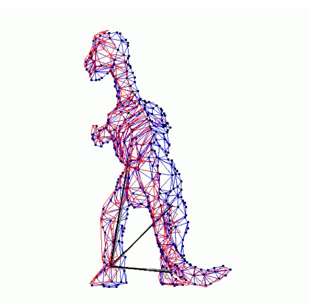
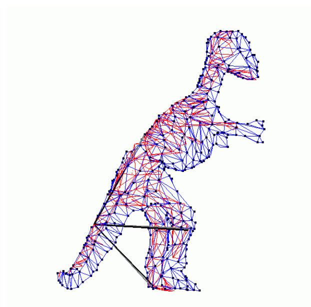
Level	Vertices of $S-$mesh	Vertices of $D-$mesh	$\mathcal{R}_y$	x	y	z	ΔR	ΔD	Time
1	3314	4152	89.8519	-0.0019	0.0004	-0.0019	0.15	0.00	67992
2	1487	1829	90.0639	-0.0006	0.0003	0.0000	0.06	0.00	13612
3	959	1135	91.1277	-0.0035	-0.0000	-0.0049	1.13	0.01	1628
4	688	855	90.0834	-0.0015	0.0000	-0.0025	0.08	0.00	831
5	426	553	91.5342	-0.0007	0.0016	-0.0019	1.53	0.00	138
6	299	395	90.2239	-0.0001	0.0002	-0.0001	0.22	0.00	45

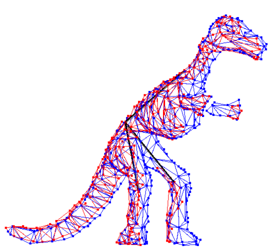
Table 3. Mesh Resolution vs Processing Time



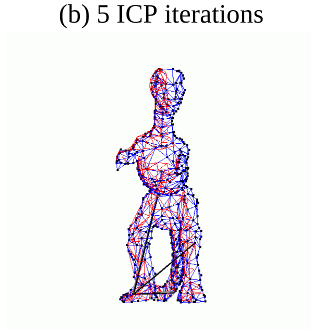
(a) 5 ICP iterations



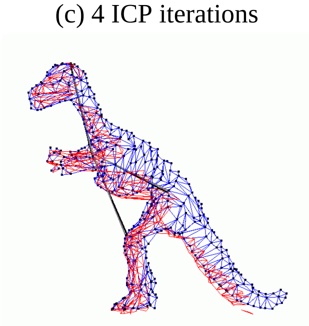
(b) 5 ICP iterations



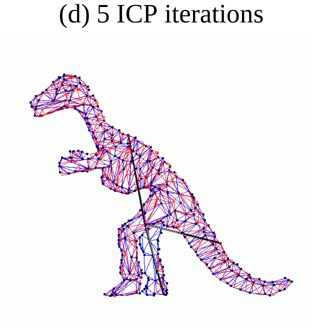
(c) 4 ICP iterations



(d) 5 ICP iterations

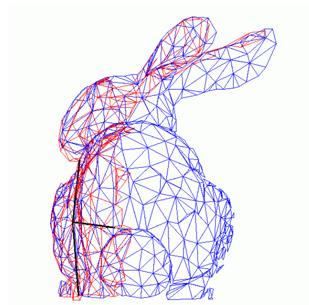


(e) 5 ICP iterations

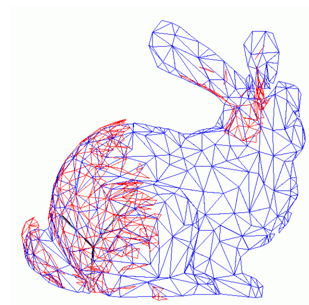


(f) 4 ICP iterations

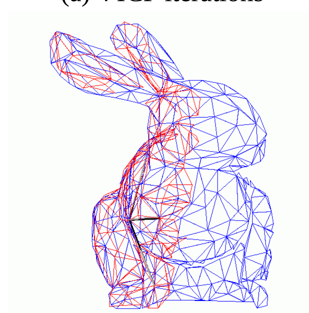
Figure 10. DINO pairs



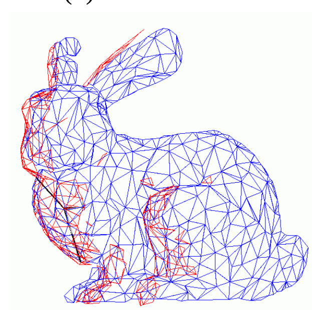
(a) 4 ICP iterations



(b) 8 ICP iterations



(c) 4 ICP iterations



(d) 5 ICP iterations

Figure 11. BUNNY pairs

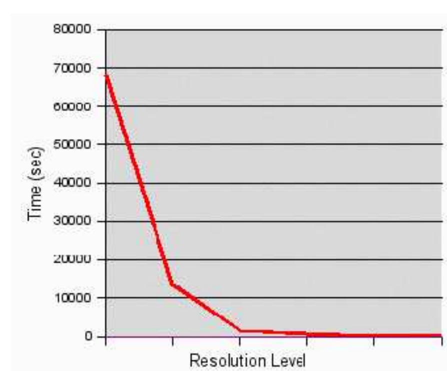


Figure 12. Resolution Level vs. Processing Time.