**98.** $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}^*.$

**99.** $2 + 5 + 8 + ... + \; ... + (2 + 3n) = \frac{(n+1)(4+3n)}{2}, \forall n \in \mathbb{N}.$

**100.** $2^0 + 2^1 + 2^2 + ... + 2^{n-1} = 2^n - 1, \forall n \in \mathbb{N}^*.$

**101.** $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}^*.$

**102.** $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2, \forall n \in \mathbb{N}^*.$

**103.** $8 \mid (3^{2n} - 1), \forall n \in \mathbb{N}^*.$

99) BÁSICA:

$n = 0$

$2 + 3 \cdot 0 = \frac{(0+1)(4+3\cdot 0)}{2}$

$2 = 2$ OK

Hipótese indutiva:

Supor que $2 + 5 + \_\_ + 2 + 3n = \frac{(n+1)(4+3n)}{2}$

Passo indutivo:

Provar que $2 + 5 \_\_ + 2 + 3n + (2 + 3(n+1))$

$= \frac{(n+1+1)(4+3(n+1))}{2}$

$$\underbrace{2 + 5 + \_ + 2 + 3n}_{\text{Por hipótese}} + 5 + 3n = \frac{(n+2)(7+3n)}{2}$$

$$\frac{(n+1)(4+3n)}{2_{/1}} + 5 + 3n_{/2} = \frac{(n+1)(4+3n) + 2(5+3n)}{2}$$

$$= \frac{1}{2}\left((n+1)(3n+4) + 6n + 10\right) = \frac{1}{2}\left[3n^2 + 4n + 3n + 4 + 6n + 10\right]$$

$$= \frac{1}{2}\left[3n^2 + 13n + 14\right]\frac{3}{3} = \frac{3}{2}\left(\frac{\cancel{3}n^2}{\cancel{3}} + \frac{13n}{3} + \frac{14}{3}\right) = \frac{3}{2}\left(n^2 + \underbrace{\frac{13n}{3}}_{S} + \underbrace{\frac{14}{3}}_{P}\right)$$

multiplicando e dividindo por 3

$\frac{42}{9} = \frac{14}{3}$

$$= \frac{3}{2}\left[\left(n + \frac{7}{3}\right)\left(n + \frac{\cancel{6}}{\cancel{3}}\right)\right] = \frac{3}{2}\left[\left(n + \frac{7}{3}\right)(n+2)\right]$$

$$= \frac{1}{2}\left(3n + \frac{\cancel{3}\cdot 7}{\cancel{3}}\right)(n+2) = \frac{1}{2}(n+2)(3n+7)$$

**98.** $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}^*.$

**99.** $2 + 5 + 8 + ... + ... + (2 + 3n) = \frac{(n+1)(4+3n)}{2}, \forall n \in \mathbb{N}.$

**100.** $2^0 + 2^1 + 2^2 + ... + 2^{n-1} = 2^n - 1, \forall n \in \mathbb{N}^*.$

**101.** $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}^*.$

**102.** $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2, \forall n \in \mathbb{N}^*.$

**103.** $8 \mid (3^{2n} - 1), \forall n \in \mathbb{N}^*.$

100) BASICA: $n=1$

$2^{(1-1)} = 2^1 - 1 \quad 1 = 1$ OK

Hipótese Indutiva:

Válido p/ $2^0 + \_ + 2^{n-1} = 2^n - 1$

Passo indutivo: Para $n+1$

Provar que $\underbrace{2^0 + \_ + 2^{n-1}}_{2^n - 1} + 2^{n+\cancel{1}-\cancel{1}} = 2^{n+1} - 1$

$2^n - 1 + 2^n =$

$= 2.2^n - 1 = 2^{n+1} - 1$, OK

101) BÁSICA: $n=1$ $\quad 1^2 = \frac{1(1+1)(2.1+1)}{6} \therefore 1 = \frac{1.\cancel{2}.\cancel{3}}{\cancel{6}} = 1$ OK

Hipótese indutiva: supor válido p/ $n$

$1^2 + 2^2 + \_ + n^2 = \frac{n(n+1)(2n+1)}{6}$

**101.** $1^2 + 2^2 + 3^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}^*.$

**102.** $1^3 + 2^3 + 3^3 + ... + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2, \forall n \in \mathbb{N}^*.$

**103.** $8 \mid (3^{2n} - 1), \forall n \in \mathbb{N}^*.$

101) BÁSICA : $n = 1$ $\quad 1^2 = \dfrac{1(1+1)(2.1+1)}{6} \therefore 1 = \dfrac{1.\cancel{2}.\cancel{3}}{\cancel{6}} = 1$ OK

Hipótese indutiva : supor válido p/ $n$

$$1^2 + 2^2 + \_ + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Passo indutivo p/ $n+1$ : Provar que $\underbrace{1^2 + 2^2 + \_ + n^2}_{} + (n+1)^2 = \dfrac{(n+1)(n+2)(2n+3)}{6}$

$$\frac{3}{2} + \frac{\cancel{4}}{\cancel{2}} = \frac{7}{2}$$

$$\frac{3}{2} + 2 = \frac{7}{2}$$

$$\frac{n(n+1)(2n+1)}{6/1} + (n+1)^2/6 = \frac{n(n+1)(2n+1) + 6(n+1)^2}{6}$$

$$\frac{(n+1)}{6}[n(2n+1) + 6(n+1)] = \frac{(n+1)}{6}[2n^2 + n + 6n + 6]$$

$$\frac{(n+1)}{6}[2n^2 + 7n + 6] \times \frac{2}{2} = \frac{2(n+1)}{6}\left[n^2 + \frac{7}{2}n + \frac{\overset{3}{\cancel{6}}}{\cancel{2}}\right]$$

$S \qquad P$

$$\frac{2(n+1)}{6}\left[\left(n + \frac{3}{2}\right)\left(n + \frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right)\right] = \frac{2(n+1)}{6}\left(n + \frac{3}{2}\right)(n+2)$$

$$= \frac{(n+1)}{6}(2n+3)(n+2) = \frac{(n+1)(n+2)(2n+3)}{6}$$