(1) $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}^*$

(2) $2 + 5 + 8 + \ldots + \ldots + (2 + 3n) = \frac{n(4+3n)}{2}, \forall n \in \mathbb{N}$

1) BÁSICA: $n = 1$ $\quad 1 = \frac{1\cancel{(1+1)}}{\cancel{2}} = 1$ OK

2) HIPÓTESE INDUTIVA: Admitir que

$S(n) = \frac{n(n+1)}{2}$. Então $S(n+1) = \frac{(n+1)(n+1+1)}{2}$

$= \frac{(n+1)(n+2)}{2}$

3) Passo Indutivo: Provar $S(n+1)$

$$\underbrace{1 + 2 + 3 + \ldots + n}_{\text{Por hipótese } \frac{n(n+1)}{2}} + n + 1 =$$

$$= \frac{n(n+1)}{2_{/1}} + \frac{n+1}{{}^{1}/_{2}} = \frac{\overbrace{n(n+1)} + 2 \cdot \overbrace{(n+1)}}{2} = \frac{(n+1)(n+2)}{2}$$, como esperado

OK

(1) $1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}, \ \forall\, n \in \mathbb{N}^*$

(2) $2 + 5 + 8 + \ldots + \ldots + (2 + 3n) = \dfrac{n(4+3n)}{2}, \ \forall\, n \in \mathbb{N}$

Vamos testar $n = 0$

$2 + 3.0 = 2$

$n = 1$

$2 + 3.1 = 5$

$n = 2$

$2 + 3.2 = 8$

2) BÁSICA: $n_0 = 0$

$(2 + 3.0) = 2 = \dfrac{0.(4 + 3.0)}{2}$

$2 = 0$?

FALSO

1) Observar

2) Perguntas
Hipótese

3) Experimentação

4) Conclusão

Não deu

Ok

ENSAIO SOBRE O MÉTODO

DESCARTES

Lógica da Pesquisa Científica

Karl Popper

(3) $2^0 + 2^1 + 2^2 + \dots + 2^{n-1} = 2^n - 1 \ \forall n \in \mathbb{N}^*$

(4) $1^2 + 2^2 + 3^2 + \dots + n^2 = \dfrac{n(n+1)(2n+1)}{6}, \ \forall n \in \mathbb{N}^*$

3) BÁSICA: $n_0 = 1$

$2^{1-1} = 2^0 = 1 = 2^1 - 1$ OK

HIPÓTESE INDUTIVA: p/ n $2^0 + \dots + 2^{n-1} = 2^n - 1$

Assim, espera-se que $\underbrace{2^0 + \dots + 2^{n-1}}_{2^n - 1 \text{ p.H.}} + 2^n = 2^{n+1} - 1$

PASSO INDUTIVO:

$$2^n - 1 + 2^n = \underbrace{2^n + 2^n}_{2 \cdot 2^n = 2^{n+1}} - 1 = 2^{n+1} - 1 \quad \text{OK}$$

4) BÁSICA: $n_0 = 1$ $\quad 1^2 = \dfrac{1(1+1)(2 \cdot 1 + 1)}{6} = 1$ OK

H.I) Admitir que $1^2 + 2^2 + \dots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Provar que $\underbrace{1^2 + 2^2 + \dots + n^2}_{} + (n+1)^2 = \dfrac{(n+1)(n+2)(2n+3)}{6}$

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = (n+1)\left(\frac{n(2n+1)}{6/1} + \frac{n+1}{1/6}\right)$$

$$= \frac{(n+1)}{6}\left(n(2n+1) + n + 1\right) = \frac{(n+1)}{6}\left(2n^2 + n + 6n + 6\right)$$

$$= \frac{(n+1)}{6}\left(2n^2 + 7n + 6\right) = \frac{(n+1)}{6} \cdot 2\left(n^2 + \frac{7}{2}n + 3\right)$$

$S = \dfrac{7}{2}$ $\quad P = \dfrac{6}{2}$ $\quad \dfrac{3}{2} + \dfrac{4}{2}$

$$= \frac{(n+1)}{6} \cdot 2\left(n + \frac{3}{2}\right)\left(n + \frac{4}{2}\right)$$

$$= \frac{(n+1)}{6} \cdot 2\left(n + \frac{3}{2}\right)(n+2)$$

$$= \frac{(n+1)}{6} \cdot (2n+3)(n+2) \quad \text{OK}$$

(5) $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2, \ \forall \ n \in \mathbb{N}^*$ $\qquad n_0 = 1$

BÁSICA: $1^3 = \left[\frac{1(1+1)}{2}\right]^2$, $1 = 1$ OK

$n = 1$

Hipótese indutiva: Admitir que

$$1^3 + 2^3 + \_\_\_ + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Logo $$1^3 + 2^3 + \_\_\_ + n^3 + (n+1)^3 = \left[\frac{(n+1)(n+2)}{2}\right]^2$$

Passo Indutivo:

$$\underbrace{1^3 + 2^3 + \_\_\_ + n^2}_{\left[\frac{n(n+1)}{2}\right]^2 \text{ p.h.i.}} + (n+1)^3$$

$$\left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3 = (n+1)^2\left[\frac{n^2}{4} + n + 1\right]$$

$$(n+1)^2\left[\frac{n^2+4n+4}{4}\right] = \frac{1}{4}(n+1)^2(n+2)^2$$

$$= \left(\frac{1}{2}\right)^2(n+1)^2(n+2)^2 = \left[\frac{1}{2}(n+1)(n+2)\right]^2 \text{ OK}$$