$3 \mid n^4 - 4n^2$

$n^4 - 4n^2 = (n^2-2n)(n^2+2n) = n^2(n-2)(n+2)$ $\quad n = \begin{cases} 3k \\ 3k+1 \\ 3k+2 \end{cases}$

$3 \mid (3k)^2, \quad 3 \mid (3k+1+2), \quad 3 \mid (3k+2-2)$

Básica: $n = 0$, $\quad 0^4 - 4\,0^2 = 0$, $\quad 3 \mid 0 \Rightarrow$ OK

H.I: Supor válido p/ $n+1 \Rightarrow 3 \mid (n+1)^4 - 4(n+1)^2$

Passo indutivo: $(n+1)^4 - 4(n+1)^2 = (n+1)^2((n+1)^2 - 4) = (n+1)^2(n^2+2n+1-4)$

$= (n+1)^2(n^2+2n-3) = (n^2+2n+1)(n^2+2n-3)$

$= n^4 + \cancel{2n^3} - 3n^2 + \cancel{2n^3} + 4n^2 - 6n + n^2 + 2n - 3$ $\quad -4n^2 \quad +4n^2$

$= n^4 - 4n^2 + 4n^3 + 6n^2 - 4n - 3$

$= n^4 - 4n^2 + 6n^2 - 3 + 4n^3 - 4n = n^4 - 4n^2 + 6n^2 - 3 + 4n(n^2-1)$

$= \underbrace{n^4 - 4n^2}_{3\mid} + \underbrace{6n^2}_{3\mid} - \underbrace{3}_{3\mid} + 4n(n-1)(n+1)$ $\quad \begin{cases} n = 3k \\ n = 3k+1 \\ n = 3k+2 \end{cases}$

$\underset{3\mid}{4\cdot(3k)}, \quad \underset{3\mid}{(3k+1-1)}, \quad \underset{3\mid}{(3k+2+1)}$

OK